Lifetime limits on (B-L)-violating nucleon decay and di-nucleon decay modes from the Fréjus experiment

Fréjus Collaboration

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The fully contained events recorded in the Fréjus detector are used to search for (B-L)-violating nucleon decay and di-nucleon decay processes. No signal is found for a sensitivity of 2.0 kiloton year. The lower limits on the partial lifetime for the various nucleon decay modes range from 5.4×10^{30} yr for $p \rightarrow \mu^- \pi^+ K^+$ to 1.0×10^{32} yr for $pn \rightarrow e^+ n$. We also quote limits on neutron and di-neutron decay into three and two neutrinos respectively.

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1. Introduction

Baryon-number violation is essential to explain the existence of matter present in the universe [1]. The definite prediction of nucleon decay in the framework of the minimal SU(5) GUT [2] has triggered considerable experimental effort [3–10]. Up to now no experimental evidence for nucleon decay has been found thus ruling out the minimal SU(5) unification scheme. In addition it has been shown that neither the minimal SU(5) nor its supersymmetric extension [11] is capable to produce the required amount

of baryon asymmetry in the GUT phase needed for the present matter density in the universe [12]. Furthermore in the big bang approach, the baryon-number-violating anomaly of the standard electroweak model may have diluted all preexisting baryon asymmetries generated by (B-L)-conserving processes as soon as the universe has cooled down to 100 GeV [13]. Within the electroweak standard model the anomaly itself is unlikely to generate the present matter density in the universe because of the smallness of CP violation [14]. To circumvent this cosmological problem either the Higgs sector of the SU(5) has to be extended or grand unification proceeds via different schemes like the partial unification in SO(10) [15]. Models of this kind would give rise to new classes of baryon-number-violating processes like nn oscillations, HH oscillations, di-nucleon decay of nucleons bound in nuclei and (B-L)-violating nucleon decay [16].

In this letter we report on new results of the search for baryon-number-violating processes obtained with the Fréjus experiment. We only consider nucleon decay modes which violate B-L as well as di-nucleon decays, including processes mediated via virtual meson exchange [17]. Results on other decay channels and $n\bar{n}$ oscillations have already been published [7,8,18]. The analysis makes especially use of the high resolution capability for multi-prong events and the low detection threshold for pions and protons in the Fréjus detector. Lower lifetime limits for 39 different baryon-number-violating channels will be given.

2. The experiment

The Fréjus detector has been described in detail in ref. [19]. The fine granularity of this 900 ton tracking calorimeter is achieved by a sandwich structure consisting of 912 flash chambers (5 mm×5 mm cells) and iron (3 mm) planes interspersed with 113 planes of Geiger tubes (15 mm×15 mm cells) which provide the trigger. The detector cells are oriented vertical and horizontal alternately, thus providing two independent orthogonal views for each event.

The trigger requires grouped hits in a small volume (1 m³) typical for nucleon decay events corresponding to an energy threshold of about 200 MeV. For the

baryon-number-violating processes investigated here this trigger threshold results in a trigger efficiency ranging from 25% for $p \rightarrow \mu^+ \nu \nu$ to 97% for $pp \rightarrow e^+ e^+$. The efficiency of the detector was constantly monitored by analyzing the atmospheric muons passing through the apparatus. The average trigger rate is 45 events per hour. Half of the triggers are due to cosmic ray muons while the rest is induced by local radioactivity and electronic noise. The event rate produced by interactions of atmospheric neutrinos is about one event per week.

3. Data processing

Continuous data taking started February 19, 1984 with a mass of 240 tons. The size of the detector was gradually increased until the final mass of 912 tons was reached in June 1985. Data taking ended September 13, 1988.

This analysis is based on contained events only. An event is defined to be contained if no prong (track or shower) is leaving the detector taking into account the detector geometry and the cell efficiency. To avoid biases due to photons leaving the detector before being converted, a fiducial cut is applied to the events. Thus a minimum distance of 25 cm for the nucleon decay vertex with respect to the surface of the detector is required. This reduces the fiducial mass to 700 tons leading to an exposure of 2.0 kiloton year (kty). With this cut we select 153 fully contained events out of which 90 events have at least two prongs.

A pattern recognition program *1 is applied to reconstruct the events. Only the vertices of the events are determined visually on a graphic terminal while track finding, association of tracks in the two independent views, particle type identification (showers or tracks) and momentum determination is done by program.

Eighteen events exhibit more than one vertex due to secondary interactions of pions. The visible energy of non-showering particles is determined by range measurement, including, in case of visible secondar-

^{*1} More details on the pattern recognition program, Monte Carlo simulation and the estimation of the neutrino induced background can be found in the description of analysis 2 in ref. [8].

ies, their kinetic energies. The shower energy is determined from the number of associated hits using the calibration obtained with the test detector [19]. For four events the correlation of tracks in the two independent views is not unique leading to different event definitions. In case of ambiguities the vertex and the event definition closest to the nucleon decay hypothesis is chosen.

4. Simulation of baryon-number-violating processes and the neutrino background

The detection efficiencies for the various baryonnumber-violating processes are determined by means of Monte Carlo methods. About 300 events are generated for each considered mode. In order to minimize the scanning bias during the interactive vertex determination the events of the different processes are mixed randomly.

The background for nucleon decay is due to atmospheric neutrino interactions and is determined using the neutrino Monte Carlo described in ref. [20]. We have simulated neutrino interactions corresponding to a sensitivity of 12.8 kty. The background neutrino events were mixed with simulated nucleon decay events and processed together imposing the nucleon decay hypothesis to all events.

The influence of the target nucleus on the hadrons produced in proton decay and in neutrino interactions is simulated by the nuclear cascade model (B) of ref. [8]. The propagation through nuclear matter of the Δ resonance formed in various nucleon decay channels with virtual meson absorption is analogous to the treatment of the Δ resonance produced by pion nucleon interaction in this cascade model.

5. Data analysis

For each decay mode we select events according to their topology taking into account the influence of nuclear effects. The accepted topologies in the analysis of a specific nucleon decay channel may differ slightly from what is expected from our knowledge of the decay products because the probability of misidentification of the particle type at very low energies amounts to about (10-20)% and no a priori as-

sumptions on possible topologies are made by the pattern recognition program.

Two different schemes were adopted for the analysis of the various nucleon decay modes. The analysis of decay channels with only one particle visible in the detector e.g. $p \rightarrow e^+ vv$ or $pn \rightarrow e^+ n$ is straightforward. In this case we require one contained prong with the correct particle type assignment by the reconstruction program. Cuts are applied to the accepted momentum range of this particle according to the expectation from nucleon decay and neutrino interaction kinematics.

For decay modes with at least two particles expected to be visible in the final state we first select events according to the accepted topologies and apply a cut in that kinematic variable leading to the best discrimination between the nucleon decay channel and the neutrino induced background. We subsequently use the method of discriminant analysis [21] to improve the separation further. Instead of applying cuts in one dimensional kinematic quantities (e.g. lepton momentum, visible energy, invariant mass etc.) this method uses the full correlations between these variables. In this method a linear combination of all kinematic variables is compared to a discriminant constant (Δ). The values of the coefficients of the linear combination are chosen to maximize the separation of two different physical processes to be disentangled. The value of Δ is determined by maximizing the ratio of ϵ/B_{90} , where ϵ is the detection efficiency for the baryon-number-violating process and B_{90} is the expected upper limit (90% CL) of the neutrino background.

6. Lifetime limits

In this study we have analyzed 35 baryon-numberviolating channels with at least one visible particle in the final state. Since no signal was observed in any of these channels only lower limits on the lifetime for the different processes can be inferred. The 90% CL lower limit on the lifetime is calculated according to

$$\frac{\tau}{\text{BR}} \geqslant AN_{\text{d}} \frac{\epsilon}{S_{90}} \,. \tag{1}$$

Herein, BR, ϵ and S_{90} denote the branching ratio, the detection efficiency and the 90% CL upper limit of a

possible signal for the investigated channel respectively, while A stands for the fiducial sensitivity which amounts to 2.0 kty in this analysis. The number of source particles for the decay (e.g. protons, neutrons or nuclei) per kiloton in our detector is given by $N_{\rm d}$. The values of $N_{\rm d}$ are summarized in table 1 for the different classes of baryon-number-violating processes considered in this paper. Since in this analysis the final state of a specific decay mode is defined uniquely, the upper limit is simply calculated from the number of candidates $N_{\rm c}$ and the expected background B according to

$$CL = 1 - \frac{\sum_{n=0}^{N_c} (n!)^{-1} (S+B)^n e^{-(S+B)}}{\sum_{n=0}^{N_c} (n!)^{-1} B^n e^{-B}}.$$
 (2)

The results on $\Delta B = 1$ nucleon decay modes with $\Delta (B-L) = 2$ and on channels mediated by virtual meson exchange $[\Delta (B-L) = 0]$ are summarized in table 2 while table 3 presents our lower lifetime limits on di-nucleon decay channels. A detailed estimate of the systematic uncertainties on the lifetime limits can be found in ref. [8]. The uncertainties are of the order of 30% in general and amount to 60% for channels dominated by pion-nucleus interactions.

Beside the decay modes with only one particle vis-

Table 1

The number of source particles for the decay (e.g. protons, neutrons or nuclei) per kiloton for the different baryon-number-violating processes analyzed in this paper. Since for the nucleon decay modes with virtual meson exchange one of the two nucleons simply acts as spectator we take the number of protons or neutrons as for (B-L)-violating nucleon decay, except for decay channels of the type $pn \rightarrow \ell^+ N$ where either the neutron or the proton may decay. We thus take the total number of nucleons in this case. The decay rate of the true di-nucleon decay modes should be strongly affected by nuclear physics. Independent of physical details the lifetime should be proportional to the nuclear density which is nearly independent of the nuclear size. We therefore use the number of iron nuclei to calculate the lower lifetime limits for di-nucleon decay channels.

Process	$\frac{N_{\rm d}}{(10^{32}{\rm kt}^{-1})}$			
p→e ⁻ X	2.80			
$n \rightarrow e^- X$	3.23			
$pp \rightarrow \ell^+ N$	2.80			
$pn \rightarrow \ell^+ N$	6.03			
$nn \rightarrow \ell^+ N$	3.23			
$NN \rightarrow \ell^+\ell$	0.11			
$NN \rightarrow \pi\pi$	0.11			

ible in the detector where the neutrino background is rather high we observe 17 independent candidates in total. This is in good agreement with the Monte Carlo prediction of 22.7 events induced by interactions of atmospheric neutrinos.

7. Neutron decay into neutrinos

To set limits on processes with only neutrinos in the final state the earth is assumed as decay target. The expected signals in the detector induced by neutrinos are single prong events which is the dominant topology at these energies with an energy spectrum similar for $n\rightarrow 3\nu$ and $nn\rightarrow 2\nu$ to the charged lepton spectrum in the decay modes $p\rightarrow \ell^+\nu\nu$ and $pn\rightarrow \ell^+\nu$ respectively. The detection sensitivity η of neutrinos stemming from these kinds of baryon number violating processes in the earth is defined as

$$\eta = AN_{\rm d}\epsilon\sigma_{\rm v}$$
(3)

where $N_{\rm d} = 6 \times 10^{32}$ yr denotes the number of nucleons per kiloton, ϵ the detection efficiency of one prong events induced by neutrinos interacting in the detector and $\sigma_{\rm v}$ the neutrino-nucleon cross section respectively. The neutrino-nucleon cross section is calculated by taking into account the contribution of neutrinos and antineutrinos produced per decay. Using the observed number of candidates as selected for the corresponding modes with one charged lepton in the final state the upper limit on the neutrino flux is simply given by

$$\Phi_{\nu} = \frac{S_{90}}{\eta} \,. \tag{4}$$

From this flux limit we obtain the lower limit on the decay modes with neutrinos only:

$$\frac{\tau_{\rm n}}{\rm Br} \geqslant \frac{n_{\rm v} n_{\rm n}}{\Phi_{\rm v} \cdot 4\pi r_{\rm e}^2},\tag{5}$$

where n_v denotes the number of neutrinos produced per decay, $n_n = 1.8 \times 10^{51}$ stands for the number of neutrons in the earth and $r_e = 6371$ km for the radius of the earth. The lower lifetime limits for neutron and di-neutron decay into neutrinos are summarized in table 4.

Table 2 Lower limits on the nucleon lifetime at 90% CL for (B-L)-violating nucleon decay and nucleon decay via virtual meson exchange. For each decay mode the detection efficiency ϵ , the expected neutrino induced background B, the number of observed candidates $N_{\rm C}$, the upper limit on the contribution of a possible signal at 90% CL S_{90} and the lower limits on the ratio of the nucleon lifetime over the unknown branching ratio into the considered decay mode without $(\tau'_{\rm N}/{\rm BR})$ and with $(\tau_{\rm N}/{\rm BR})$ background subtraction is given. If no background events are selected from the simulated neutrino sample (with a sensitivity of 12.8 kty) a value of B < 0.1 is given corresponding to an upper limit of 66% CL on this background.

$\Delta(B-L)$	$\Delta B = 1$	<i>€</i> (%)	В	$N_{ m C}$	S_{90}	$\frac{\tau_N'/BR}{(10^{31} \text{ yr})}$	$\frac{\tau_{\rm N}/{\rm BR}}{(10^{31}{\rm yr})}$
2	$n \rightarrow \gamma \nu$	32.3	6.86	10	8.76	1.4	2.4
	$n\rightarrow e^-e^+\nu$	26.5	< 0.10	0	2.30	7.4	7.4
	$n\rightarrow e^-\mu^+\nu$	17.0	< 0.10	0	2.30	4.7	4.7
	$n \rightarrow \mu^- \mu^+ \nu$	15.1	1.40	0	2.30	4.2	4.2
	$p \rightarrow e^+ vv$	21.1	6.08	11	10.57	0.7	1.1
	$p \rightarrow \mu^+ \nu \nu$	16.2	11.23	7	4.34	0.8	2.1
	$n\rightarrow e^-\pi^+$	19.8	1.09	0	2.30	5.5	5.5
	$n \rightarrow \mu^- \pi^+$	11.7	1.40	0	2.30	3.3	3.3
	$n \rightarrow e^{-}K^{+}$	21.7	2.96	3	4.38	2.1	3.2
	$n \rightarrow \mu^- K^+$	20.4	2.18	0	2.30	5.7	5.7
	$p\rightarrow e^-\pi^+\pi^+$	15.9	2.50	1	2.91	2.3	3.0
	$p\rightarrow \mu^-\pi^+\pi^+$	9.4	1.72	1	3.06	1.4	1.7
	$n \rightarrow e^- \pi^+ \pi^0$	15.0	0.78	1	3.36	2.5	2.9
	$n\rightarrow \mu^-\pi^+\pi^0$	12.1	0.78	0	2.30	3.4	3.4
	$p\rightarrow e^{-}\pi^{+}K^{+}$	16.5	2.50	3	4.62	1.4	2.0
	$p\rightarrow \mu^-\pi^+K^+$	4.5	0.78	2	4.61	0.5	0.5
0	pn→e ⁺ n	42.3	6.40	5	4.56	5.0	10.3
	pn→μ ⁺ n	52.0	5.93	7	6.42	5.0	8.9
	pp→e ⁺ p	33.7	0.16	0	2.30	8.2	8.2
	$pp \rightarrow \mu^+ p$	43.4	2.96	2	3.53	4.5	6.8
	pp→e ⁺ Δ ⁺	23.1	1.40	1	3.14	3.3	4.1
	$pp \rightarrow \mu^+ \Delta^+$	21.5	4.84	2	3.15	2.3	3.8
	$pn \rightarrow e^+ \Delta^0$	16.5	1.25	1	3.19	4.8	5.7
	$pn \rightarrow \mu^+ \Delta^0$	19.1	6.86	2	2.93	4.0	7.1
	$nn \rightarrow e^+\Delta^-$	14.2	2.96	2	3.53	1.7	2.6
	nn→μ+Δ-	20.4	6.71	4	3.86	1.7	3.4

Table 3
Lower limits on the lifetime per nucleus at 90% CL for di-nucleon decay. (For details on the information given in this table see table 2.) Since the limits are given per nucleus the same results as for $pp \rightarrow \ell^+\ell^-$ are obtained for $nn \rightarrow \ell^+\ell^-$.

$\Delta(B-L)$	$\Delta B = 2$	<i>ϵ</i> (%)	В	$N_{ m C}$	S_{90}	τ'_{N}/BR (10 ³⁰ yr)	$\frac{\tau_{\rm N}/{\rm BR}}{(10^{30}{\rm yr})}$
2	$pp \rightarrow \pi^+\pi^+$	19.0	2.34	4	5.81	0.5	0.7
	$pn \rightarrow \pi^+\pi^0$	23.4	0.31	0	2.30	2.0	2.0
	$nn \rightarrow \pi^0 \pi^0$	36.7	0.78	0	2.30	3.4	3.4
	$nn \rightarrow \pi^+\pi^-$	19.8	2.18	4	5.94	0.5	-0.7
0	pp→e ⁺ e ⁺	62.3	< 0.10	0	2.30	5.8	5.8
	$pp \rightarrow e^+ \mu^+$	38.8	< 0.10	0	2.30	3.6	3.6
	$pp \rightarrow \mu^{+}\mu^{+}$	18.4	0.62	0	2.30	1.7	1.7
	$pn \rightarrow e^+ \bar{v}$	52.8	9.67	5	3.77	1.1	2.8
	$pn \rightarrow \mu^+ \bar{v}$	37.4	4.37	4	4.61	0.9	1.6

Table 4
Lower limits on the nucleon lifetime at 90% CL for neutron and di-neutron decay into neutrinos. Φ_v is the upper limit on the measured neutrino flux for the selected neutrino type and η the detection sensitivity. (For details on the information given in the other columns see table 2.) For the di-neutron decay we calculate the lifetime limit per nucleus assuming a mean number of neutrons of 20 per nucleus in the earth. Lifetime limits for decay modes with a different combination of neutrinos and antineutrinos may be obtained from these limits by multiplying with the ratio of averaged cross sections.

$\Delta(B\!-\!L)\!=\!2$	$\Phi_{\rm v}$ (s ⁻¹ cm ⁻²)	η (s cm²)	В	$N_{ m C}$	S_{90}	$\tau'_{\rm N}/{\rm BR}$ (10 ²⁵ yr)	$\frac{\tau_{\rm N}/{\rm BR}}{(10^{25}{\rm yr})}$
$n \rightarrow v_e v_e \bar{v}_e$	1.1	9.8	6.1	11	10.6	2	3
$n \rightarrow \nu_{\mu} \nu_{\mu} \bar{\nu}_{\mu}$	0.28	15.7	11.2	7	4.3	5	12
$nn \rightarrow v_e \bar{v}_e$	0.10	39	9.7	5	3.8	0.5	1.2
$nn \rightarrow \nu_{\mu} \bar{\nu}_{\mu}$	0.18	26	4.4	4	4.6	0.4	0.6

8. Conclusions

We have analyzed the data recorded in the Fréjus detector corresponding to a sensitivity of 2.0 kty in order to search for (B-L)-violating nucleon decay and di-nucleon decay modes. No signal was found in any of the 39 different channels investigated.

For the $\Delta B = 1$ decay modes with visible particles in the final state the lower limits on the partial lifetime range from 5.4×10^{30} to 1.0×10^{32} yr. In this sample of decay channels all $\Delta(B-L)=0$ modes as well as six $\Delta(B-L)=2$ modes are presented for the first time. Only for ten decay modes have limits already been published by other experiments. Although the water Čerenkov detectors, IMB [4] and Kamiokande [5] have reached higher sensitivities, only for five of the decay modes considered here have results already been published by IMB with values comparable to our limits. The HPW experiment [9] sets lower limits on seven decay channels in common with our analysis which are about one order of magnitude smaller than our results. For $\Delta B = 2$ processes, the lower limits on the iron nucleus lifetime range from 0.7 to 5.8×10^{30} yr and are presented for the first time except for the mode pp→e+e+ for which NU-SEX [6] gave a limit which has been improved by one order of magnitude.

Using the earth as decay target we also set lower lifetime limits on neutron and di-neutron decay into neutrinos. These limits are based on the measurement of the neutrino flux for ν_e and ν_μ separately in the energy range of interest $^{\sharp 2}$.

Unfortunately for the channels investigated here theoretical predictions are so far not as firm as were the minimal SU(5) predictions for the $\Delta(B-L)=0$ modes since other grand unification schemes like SO(10) contain too many unknown parameters.

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^{**2} The sensitivity to this kind of nucleon decay is simply limited by the atmospheric neutrino flux and thus is nearly independent of the number of kty collected by the different experiments. Limits on some of these decay channels were already reported in literature. Two different approaches were used to derive limits. One possibility is the radiochemical method [22] where the inferred limit depends strongly on details of the nuclear model involved. Secondly one uses the horizontal muon flux measured deep under ground [23,24] to extrapolate to the ν_μ flux in the energy region of interest. However, using the muon flux quoted in ref. [24] the lifetime limit turns out to be smaller by more than one order of magnitude compared to the value given in ref. [23].

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