

PCAC and coherent pion production by neutrinos

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Abstract. Coherent π^+ and π^0 production in low energy neutrino reactions is discussed in the framework of the partially conserved axial vector current theory (PCAC). The role of lepton mass effects in suppressing the π^+ production is discussed. Instead of using models of pion nucleus scattering, the available data on pion carbon scattering are implemented for an analysis of the PCAC prediction. Our results agree well with the published upper limits for π^+ production but are much below the recent MiniBooNE result for π^0 production.

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PCAC AND FORWARD LEPTON THEOREM

We discuss single pion production in coherent charged current (CC) and neutral current (NC) reactions e.g. $\nu_\mu + {}^{12}\text{C} \rightarrow \nu_\mu + {}^{12}\text{C} + \pi^0$. Our starting point is the general formula for neutrino scattering off a nucleus or nucleon at rest

$$\frac{d\sigma^{\text{CC}}}{dQ^2 dy} = \frac{G_F^2 \cos^2 \theta_C}{4\pi^2} \kappa E \frac{Q^2}{|\mathbf{q}|^2} [u^2 \sigma_L + v^2 \sigma_R + 2uv \sigma_S] \quad (1)$$

already derived by Lee and Yang in 1962 [1] for zero mass of the outgoing lepton. The momentum and energy transfer between incoming neutrino and outgoing lepton is given by \mathbf{q} and $\nu = E - E'$. As usual $Q^2 = -q^2$ denotes the four-momentum transfer squared¹. For $Q^2 \rightarrow 0$ only the term containing the scalar cross section σ_S survives. Here Adler's forward scattering theorem [2] based on PCAC predicts

$$\sigma_{S, \nu N \rightarrow \nu' F}(W) = \frac{|\mathbf{q}|}{\kappa Q^2} f_\pi^2 \sigma_{\pi N \rightarrow F}(W) \quad (2)$$

resulting in

$$\left. \frac{d\sigma^{\text{CC}}}{dQ^2 dy} \right|_{Q^2 \rightarrow 0} = \frac{G_F^2 \cos^2 \theta_C f_\pi^2 E}{2\pi^2 |\mathbf{q}|} uv \sigma_{\pi^+ N}(W) \quad (3)$$

and²

$$\left. \frac{d\sigma^{\text{NC}}}{dQ^2 dy} \right|_{Q^2 \rightarrow 0} = \frac{G_F^2 f_\pi^2 E}{4\pi^2 |\mathbf{q}|} uv \sigma_{\pi^0 N}(W) \quad (4)$$

For CC the limit $Q^2 = 0$ cannot be reached. Therefore and for comparison with experiments we extrapolate to finite values of Q^2 by introducing a formfactor $G_A = m_A^2 / (Q^2 + m_A^2)$. In addition we include a correction (already contained in Adler's paper) due to the nearby pion pole in the hadronic axial vector current [3]

$$\frac{d\sigma^{\text{CC}}}{dQ^2 dy} = \frac{G_F^2 \cos^2 \theta_C f_\pi^2 E}{2\pi^2 |\mathbf{q}|} uv \left[\left(G_A - \frac{1}{2} \frac{Q_{\text{min}}^2}{Q^2 + m_\pi^2} \right)^2 + \frac{y}{4} (Q^2 - Q_{\text{min}}^2) \frac{Q_{\text{min}}^2}{(Q^2 + m_\pi^2)^2} \right] \sigma_{\pi^+ N} \quad (5)$$

With $Q_{\text{min}}^2 = m_\nu^2 y / (1 - y)$ the pion pole term vanishes for $m_\nu = 0$, it is a lepton mass correction.

¹ $Q^2 = \mathbf{q}^2 - \nu^2$; $y = \nu/E$; $\kappa = (W^2 - M_N^2)/2M_N$; $u, v = (E + E' \pm |\mathbf{q}|)/2E$. G_F and θ_C are the Fermi coupling constant and the Cabbibo angle.

² $f_\pi = \sqrt{2} f_{\pi^0} = 130.7 \text{ MeV}$

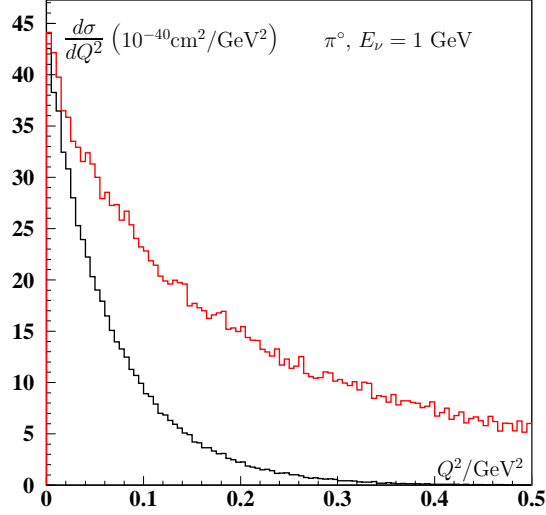


FIGURE 1. π^0 production by neutrino scattering on carbon. Black histogram calculated from integrating (7) over (t, y) , red histogram from integrating (4) over y . A hadronic toy model is used (see text).

COHERENT SCATTERING

Coherent pion nucleus (πN) scattering is strongly peaked in forward direction distinguishing it from incoherent background. We therefore expect coherent single pion production by neutrinos to be well described by the PCAC ansatz. Like in the original Rein Sehgal (RS) paper [4] this approximation is assumed to hold also for the differential cross section

$$\frac{d\sigma^{CC}}{dQ^2 dy dt} = \frac{G_F^2 \cos^2 \theta_C f_\pi^2 E}{2\pi^2 |\mathbf{q}|} uv \left[\left(G_A - \frac{1}{2} \frac{Q_{\min}^2}{Q^2 + m_\pi^2} \right)^2 + \frac{y}{4} (Q^2 - Q_{\min}^2) \frac{Q_{\min}^2}{(Q^2 + m_\pi^2)^2} \right] \frac{d\sigma(\pi^+ N \rightarrow \pi^+ N)}{dt} \quad (6)$$

and

$$\frac{d\sigma^{NC}}{dQ^2 dy dt} = \frac{G_F^2 f_\pi^2 E}{4\pi^2 |\mathbf{q}|} uv \frac{d\sigma(\pi^0 N \rightarrow \pi^0 N)}{dt} . \quad (7)$$

This extension is by no means trivial. t is the four momentum transfer squared between the incoming virtual boson and the outgoing pion. Therefore $t = 0$ cannot be reached. $t_{\min} = f(Q^2)$ results in a very effective Q^2 cutoff for exponentially decreasing hadronic differential cross sections. The t -integral of e.g. (6) approaches (5) only for $Q^2 \rightarrow 0$. Figure(1) shows as example π^0 production on carbon for $E = 1$ GeV. A hadronic toy model $d\sigma/dt = a \exp(-bt)$ with constant coefficients $a = 3200 \text{ mb/GeV}^2$, $b = 40 \text{ GeV}^{-2}$ is used.

The RS paper [4] evaluates the kinematical factor always at $Q^2 = 0$, i.e. $Euv/|\mathbf{q}| \rightarrow (1-y)/y$. At high energies the differences are negligible. At threshold they are very important. This is demonstrated in figure (2a) using the hadronic toy model. The CC/NC ratio shown in figure (2b) approaches the limiting value of $2 \cos^2 \theta_C$ also only at high energies.

THE ELASTIC PION NUCLEUS CROSS SECTION

A simple model for elastic pion nucleus scattering, which can be easily implemented into MC generators is also contained in [4]. We discuss it here for isoscalar targets of atomic mass A leading apart from electromagnetic corrections to identical cross sections for $\pi^{\pm,0}$. Starting from

$$\frac{d\sigma(\pi N \rightarrow \pi N)}{dt} = A^2 \frac{d\sigma_{\text{el}}}{dt} \Big|_{t=0} e^{-b_{\text{RS}} t} F_{\text{abs}} \quad (8)$$

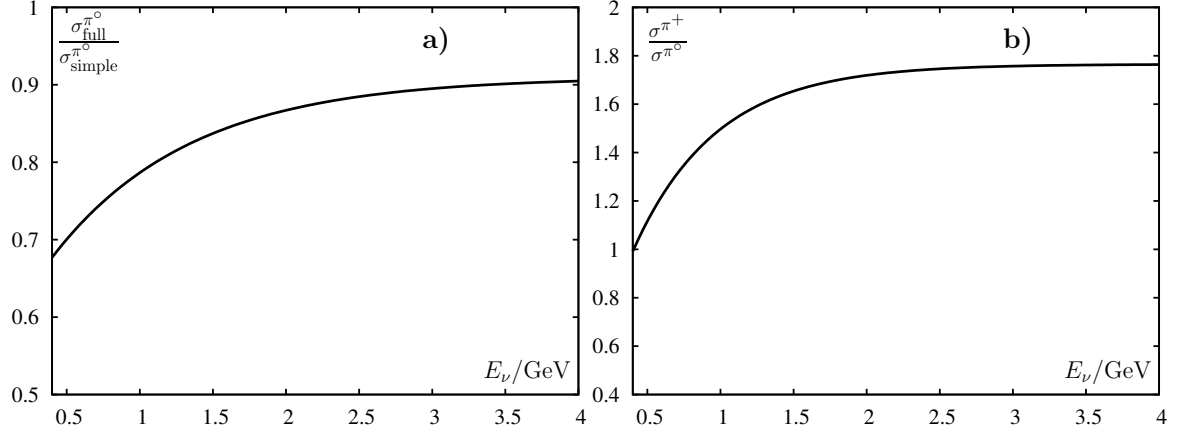


FIGURE 2. a) Ratio $\sigma_{\text{full}}^{\pi^0}/\sigma_{\text{simple}}^{\pi^0}$ of the integrated cross sections of (7) versus the energy of the incoming neutrino. $\sigma_{\text{simple}}^{\pi^0}$ is calculated using the kinematical approximations of [4]. b) Ratio $\sigma^{\pi^+}/\sigma^{\pi^0}$ of the integrated cross sections versus the energy of the incoming neutrino.

the elastic differential pion nucleon cross section at $t = 0$ is calculated with the help of the optical theorem

$$\left. \frac{d\sigma_{\text{el}}}{dt} \right|_{t=0} = \frac{1}{16\pi} \left(\frac{\sigma_{\text{tot}}^{\pi^+p} + \sigma_{\text{tot}}^{\pi^-p}}{2} \right)^2 \quad (9)$$

where the total pion proton cross sections are taken from data. The slope b_{RS} is determined via the optical model relation

$$b_{\text{RS}} = \frac{1}{3} R_0^2 A^{2/3} \text{ e.g. } R_0 = 1.057 \text{ fm} . \quad (10)$$

Finally using a simple geometrical picture the absorption factor

$$F_{\text{abs}} = \exp \left(-\frac{9A^{1/3}}{16\pi R_0^2} \sigma_{\text{inel}} \right) \quad (11)$$

is calculated from data for inelastic pion proton scattering via

$$\sigma_{\text{inel}} = \frac{\sigma_{\text{inel}}^{\pi^+p} + \sigma_{\text{inel}}^{\pi^-p}}{2} . \quad (12)$$

Although this model has its limitations (e.g. it predicts $d\sigma/dt \rightarrow 0$ for $A \rightarrow \infty$) it has been very successful in describing high energy coherent neutrino scattering [3].

In order to obtain a more precise prediction for the pion nucleus cross section in the resonance region the parameterization of the pion nucleon cross sections used in RS [4] has been replaced by detailed fits to the $\pi^\pm p$ data published by the Particle Data Group [5]. An example is shown in figure (3a). The curve labelled RS2009 in figure (3b) shows the resulting total CC coherent neutrino cross section for energies up to 2 GeV. There exist various implementations of the RS-model which, however, obtain different results. An example is displayed in figure (3b). The predictions of other Monte Carlo generators claiming to use the RS-model show even more pronounced discrepancies [7]. The reason for these differences remains a puzzle.

For the resonance region with its rapidly varying cross sections and angular distributions the hadronic RS-model is probably too simple. It describes badly the low energy experimental data on elastic $\pi^{12}\text{C}$ scattering. Instead of refining it we – in the spirit of Adler’s theorem – directly revert to the measured $\pi^{12}\text{C}$ cross sections. Pion carbon scattering data with $30 < T_\pi < 776 \text{ MeV}$ of various experiments have been subjected to a phase shift analysis and extrapolated to $T_\pi = 870 \text{ MeV}$ by the Karlsruhe group [8]. Figure (4a) shows an example for the differential cross section $d\sigma/dt$ reconstructed from these phaseshifts at $T_\pi = 162 \text{ MeV}$, close to the maximum of the first resonance. The forward scattering containing the bulk of the cross section is then fitted by a $a \exp(-bt)$ ansatz resulting in energy dependent coefficients a, b [9]. Using this parameterization the pion carbon elastic cross section in the resonance region is below the hadronic RS-model but approaches it quickly at higher p_π , see figure (4b).

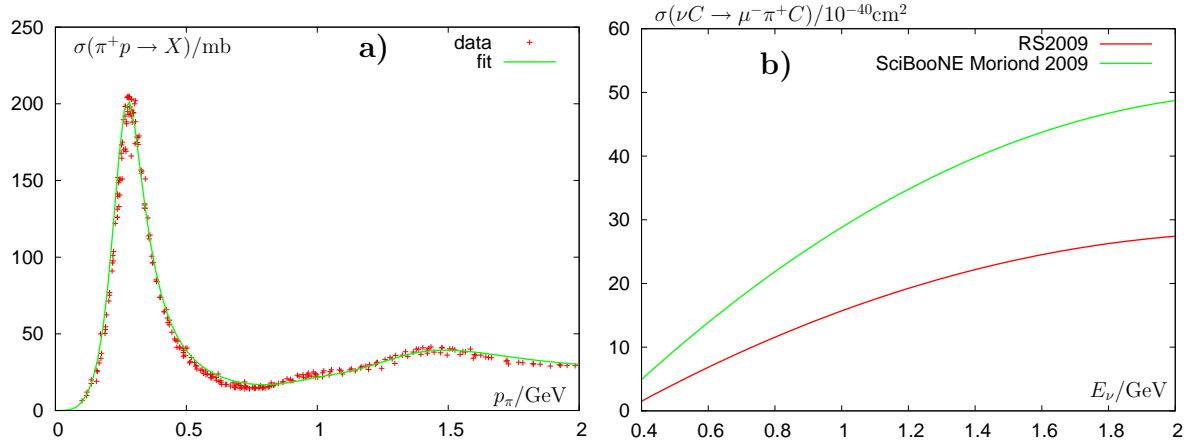


FIGURE 3. a) Fit to the total $\pi^+ p$ cross section using the PDG tables [5] for pion laboratory momenta up to 2 GeV. b) Total CC cross section versus neutrino energy using the updated hadronic RS model (RS2009). For comparison the prediction used by the SciBooNE Collaboration [6] is also shown.

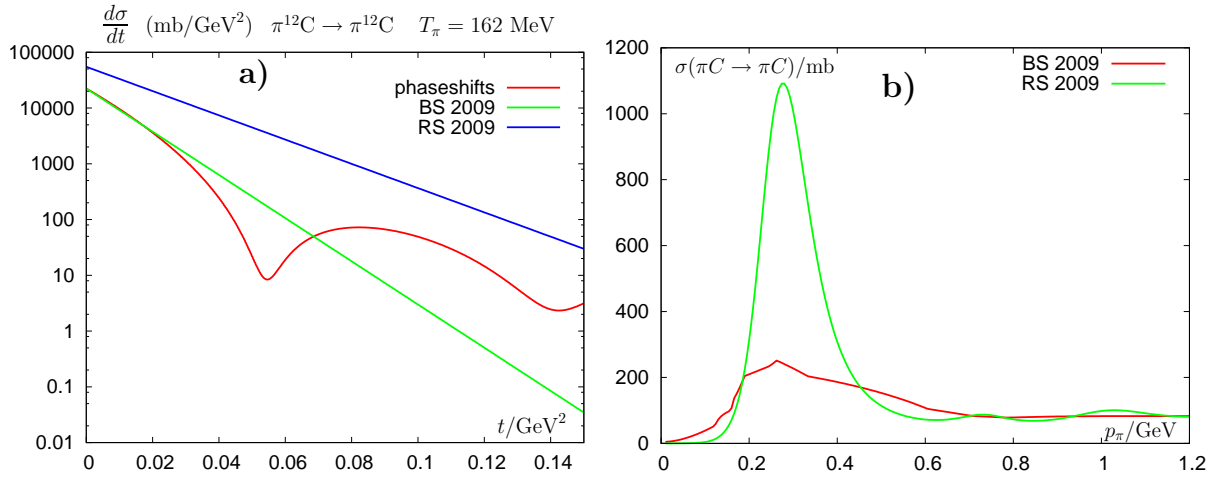


FIGURE 4. a) Differential cross section $d\sigma/dt$ for elastic pion carbon scattering. The red line labelled 'phaseshifts' is calculated from the phaseshifts in [8]. The green line (BS2009) is an exponential fit to the forward cross section. The blue line (RS2009) represents the result of the updated RS hadronic model. b) Total elastic pion carbon cross section versus the laboratory pion momentum in the updated RS hadronic model (RS2009) and from exponential fits to the phaseshift analysis (BS2009).

RESULTS

Using the fits discussed in the preceding section we get a substantial modification of the PCAC prediction for pion production off carbon nuclei for NC and CC reactions at low neutrino energies. This is demonstrated in figure (5) where the new results are compared with calculations using the updated hadronic RS-model.

Our predictions for the total cross section are compatible with other PCAC based calculations [10] and remarkably close to certain variants of microscopic nuclear physics models [11, 12]. Differential distributions are more sensitive to model details. The MiniBooNE collaboration has proposed to use $E_\pi(1 - \cos \Theta_\pi)$ as variable for the analysis of neutrino scattering [13] (E_π and Θ_π defined in the laboratory system). As can be seen in figure (6) a recent nuclear physics model [14, 12] agrees well with our PCAC model at a neutrino energy typical for the MiniBooNE experiment.

The extension of the new ansatz to other nuclei is of particular importance. At this moment we propose to use in the spirit of the optical model an $A^{2/3}$ scaling law which is close to the effective A -dependence obtained in the hadronic RS-model for light nuclei.

Our results agree with the published experimental limits on coherent π^+ production [15, 16]. Using the $A^{2/3}$

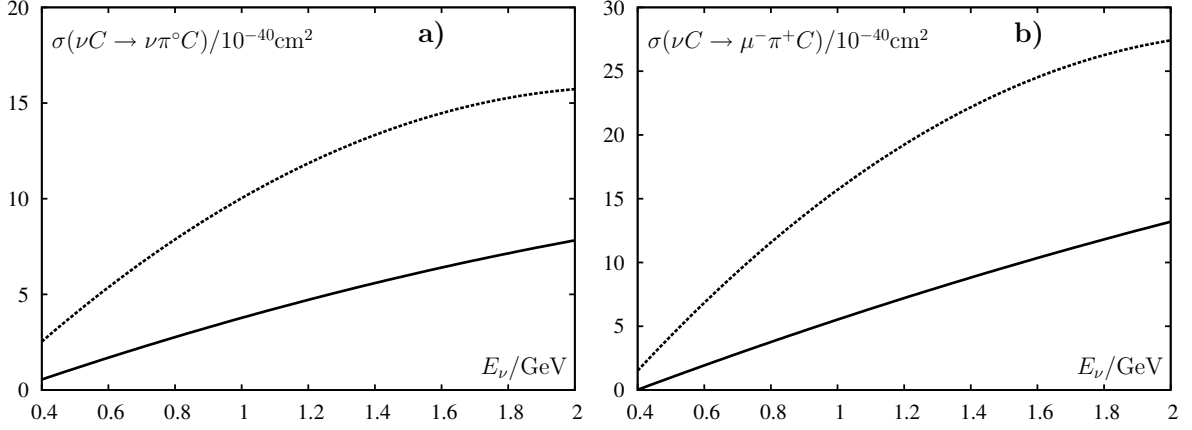


FIGURE 5. Cross section per nucleus of coherent π production by neutrinos off carbon nuclei, a) NC reaction $\nu_\mu + {}^{12}\text{C} \rightarrow \nu_\mu + {}^{12}\text{C} + \pi^0$, b) CC reaction $\nu_\mu + {}^{12}\text{C} \rightarrow \mu^- + {}^{12}\text{C} + \pi^+$. The data in units of 10^{-40} cm^2 are plotted versus the neutrino energy in GeV. The upper curve is calculated using the hadronic RS model, the lower curve using our parametrization of pion carbon scattering data.

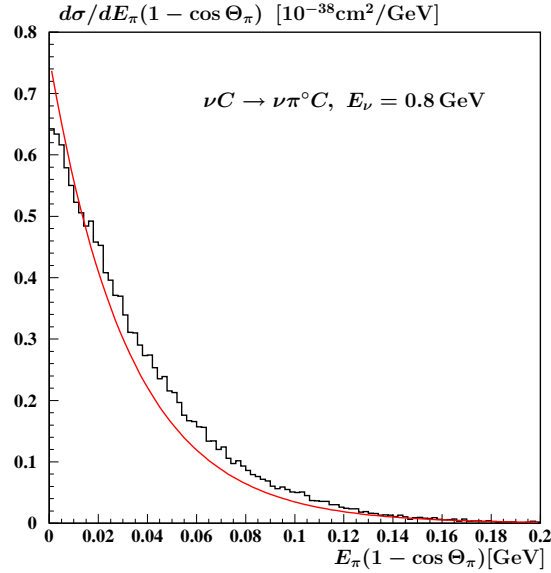


FIGURE 6. Differential cross section $d\sigma/dE_\pi(1 - \cos\Theta_\pi)$ versus $E_\pi(1 - \cos\Theta_\pi)$ calculated for NC pion production at a neutrino energy of 0.8 GeV. The histogram uses the BS2009-model [9] and the curve represents the results of a recent nuclear physics calculation [14].

scaling law it also agrees with the π^0 data of the Aachen Padova experiment [17]. Like all other recent theoretical predictions we have a problem with the MiniBooNE π^0 result [13]. PCAC models have very little flexibility in tuning the predictions. Before, however, claiming that the model is falsified one would like to clarify several questions, e.g. the dependence of the experimental result on the use of inappropriate Monte Carlo models and how the experiments ensure the coherence of the process.

Instead of using a two parameter fit of the pion carbon differential cross section a variant of the new PCAC model has been studied in which the full angular distribution as represented by the phase shifts (see figure (4a)) is utilized. For energies between two data sets a linear extrapolation of the differential cross sections $d\sigma/dt$ at a fixed scattering angle in the CMS system is applied. The resulting total neutrino cross section and the pion angular distribution in the forward direction changes only at the level of a few percent. In contrast to the simpler model there is, however, a long tail at larger angles. An example is shown in figure (7). These differences might become important when precise

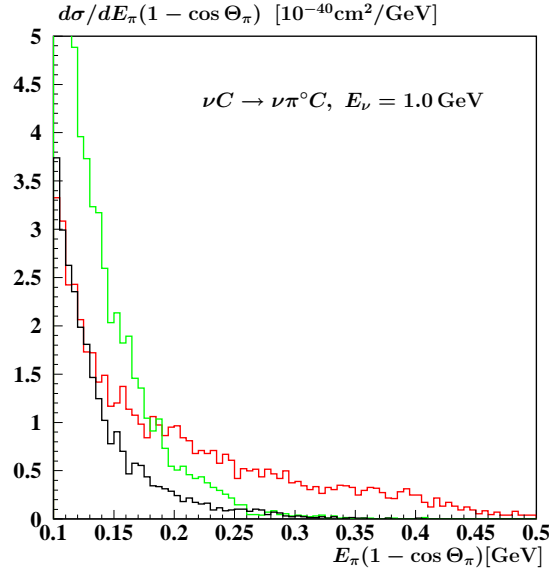


FIGURE 7. Tail of the differential cross section $d\sigma/dE_\pi(1 - \cos \Theta_\pi)$ versus $E_\pi(1 - \cos \Theta_\pi)$ calculated for NC pion production at a neutrino energy of 1.0 GeV. The black histogram is obtained using the two parameter fit to pion carbon scattering described in [9]. The red histogram uses the full angular distribution. For comparison a prediction using the updated hadronic RS-model is also shown (green histogram).

experimental data are available. With only a few sets of phaseshifts for other nuclei on-hand the extension of this model version to non carbonic targets requires further research.

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