Partially conserved axial vector current and coherent pion production by low energy neutrinos

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Coherent π^+ and π° production in low energy neutrino reactions is discussed in the framework of the partially conserved axial vector current theory . The role of lepton mass effects in suppressing the π^+ production is emphasized. Instead of using models of pion nucleus scattering, the available data on pion carbon scattering are implemented for an analysis of the partially conserved axial vector current theory prediction. Our results agree well with the published upper limits for π^+ production but are much below the recent MiniBooNE result for π° production.

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I. INTRODUCTION

The availability of high intensity neutrino beams with energies up to a few GeV opens the way to precise investigation of neutrino oscillations. Essential for these experiments is a detailed understanding of all low energy neutrino reactions especially single pion production in charged current (CC) and neutral current (NC) reactions. Coherent pion production off nuclei, e.g. in $\nu_\mu + {}^{12}\mathrm{C} \to \nu_\mu + {}^{12}\mathrm{C} + \pi^\circ$ constitutes an especially interesting subsample not only because it is a significant background to the $\nu_\mu \to \nu_e$ oscillation search but also because it is deeply rooted in fundamental physics via Adler's partially conserved axial vector current (PCAC) theorem [1,2] which connects forward neutrino scattering with the pion nucleon cross section.

II. PCAC AND FORWARD LEPTON THEOREM

Our starting point is the general expression for inelastic neutrino scattering[1]

$$\frac{d\sigma^{\mathrm{CC}}}{dQ^2dy} = \frac{G_F^2\cos^2\theta_C}{4\pi^2}\kappa E\frac{Q^2}{|\boldsymbol{q}|^2}[u^2\sigma_L + v^2\sigma_R + 2uv\sigma_S] \qquad (1)$$

already derived by Lee and Yang [3] in 1962 for zero mass of the outgoing lepton. The momentum and energy transfer between incoming neutrino and outgoing lepton is given by $\boldsymbol{q}$ and $\nu = E - E'$ with $y = \nu/E$. As usual $Q^2 = -q^2$ denotes the four-momentum transfer squared $Q^2 = \boldsymbol{q}^2 - \nu^2$ while $\kappa = (W^2 - M_N^2)/2M_N$ for a hadronic system with invariant mass W emerging from a nucleus or nucleon with mass M_N. The kinematical factors u, v are given by $u, v = (E + E' \pm |\boldsymbol{q}|)/2E$.

The cross sections $\sigma_{L,R,S}$ (where L, R, S stands for *left, right, scalar*) are unknown functions of Q^2 and W which have to be measured or calculated in theoretical models. It is well-known that for $Q^2 \to 0$ only the term with σ_S in (1) survives because the overall factor Q^2 is compensated by a factor $1/Q^2$ in the scalar cross section. In this limit PCAC predicts [4]

$$\sigma_S = \frac{|\boldsymbol{q}|}{\kappa Q^2}f_\pi^2\sigma_{\pi N}. \qquad (2)$$

Here f_π is the pion decay constant (130.7 MeV) and $\sigma_{\pi N}(W)$ the pion nucleon (or nucleus) cross section for the hadronic final state under consideration.

The resulting formula for forward inelastic neutrino scattering is[2]

$$\left.\frac{d\sigma^{\mathrm{CC}}}{dQ^2dy}\right|_{\theta_l\to 0} = \frac{G_F^2\cos^2\theta_C f_\pi^2}{2\pi^2}\frac{E}{|\boldsymbol{q}|}uv\sigma_{\pi N}(W). \qquad (3)$$

In the presence of lepton mass m_l there is a correction to this formula caused by the pion pole term in the hadronic axial vector current. Including, in addition, an axial vector form factor $G_A(Q^2)$ to describe the variation of the cross section at small values of Q^2 around the forward direction, one obtains the result given by Kopeliovich and Marage [5]

$$\frac{d\sigma^{\mathrm{CC}}}{dQ^2dy} = \frac{G_F^2\cos^2\theta_C f_\pi^2}{2\pi^2}\frac{E}{|\boldsymbol{q}|}uv\bigg[\bigg(G_A - \frac{1}{2}\frac{Q^2_{\mathrm{min}}}{Q^2 + m_\pi^2}\bigg)^2 + \frac{y}{4}(Q^2 - Q^2_{\mathrm{min}})\frac{Q^2_{\mathrm{min}}}{(Q^2 + m_\pi^2)^2}\bigg]\sigma_{\pi N}, \qquad (4)$$

where $Q^2_{\mathrm{min}} = m_l^2 y/(1 - y)$ is the high energy approximation to the true minimal Q^2. The axial vector form factor G_A is defined by

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[1] G_F and θ_C are the Fermi coupling constant and the Cabbibo angle.

[2] θ_l denotes the laboratory angle of the outgoing lepton.

$$G_A = \frac{m_A^2}{Q^2 + m_A^2} \quad (5)$$

with a typical value for the axial vector meson mass m_A of 0.95 GeV; see however the extensive discussion in the literature [6].

The first term inside the rectangular brackets of (4) corresponds to outgoing muons with negative helicity (helicity nonflip) whereas the second term is the helicity flip contribution which vanishes at 0° scattering angle. This helicity structure was also found by Adler [2] and Piketty and Stodolsky [7]. With $G_A = 1$ the expression inside the rectangular brackets represents the *Adler screening factor* which was invoked in [8,9] as a possible explanation of the dip in CC reactions at low Q^2, resulting from the destructive interference of the pion pole.

For neutrino scattering off nuclei N the coherent pion channel $\nu_\mu N \to \mu^- \pi^+ N$ has special interest, since the PCAC formula (4) predicts the cross section to be proportional to the elastic cross section $\pi N \to \pi N$. This hadronic process is strongly enhanced in the forward direction because of the coherent action of the A nucleons in the nucleus, with an amplitude $\sim A$ at 0° scattering angle. This effect then implies the enhancement of forward going pions in the neutrino reaction, which is a characteristic signature of coherent pion production, distinguishing it from possible incoherent backgrounds. An early analysis of pion production by neutrinos, that included a discussion of the kinematical region termed coherent, was given in [10].

III. APPLICATION TO COHERENT PION PRODUCTION

In the following we investigate in more detail coherent single pion production. Assuming that the derivation given above also holds for the differential cross section one gets for the CC reaction $\nu_\mu N \to \mu^- \pi^+ N$

$$\frac{d\sigma^{\rm CC}}{dQ^2 dy dt} = \frac{G_F^2 \cos^2\theta_C f_\pi^2}{2\pi^2} \frac{E}{|\boldsymbol{q}|} uv \left[\left(G_A - \frac{1}{2} \frac{Q^2_{\rm min}}{Q^2 + m_\pi^2} \right)^2 + \frac{y}{4}(Q^2 - Q^2_{\rm min}) \frac{Q^2_{\rm min}}{(Q^2 + m_\pi^2)^2} \right] \times \frac{d\sigma(\pi^+ N \to \pi^+ N)}{dt}, \quad (6)$$

where t is the modulus of the four-momentum transfer squared between incoming virtual boson and outgoing pion. For calculating the NC reaction $\nu N \to \nu \pi^\circ N$ one has to set $m_l = 0$, $\theta_C = 0$ and divide the right-hand side of the resulting equation by 2, because $f_{\pi^\circ} = f_\pi/\sqrt{2}$. We thus obtain

$$\frac{d\sigma^{\rm NC}}{dQ^2 dy dt} = \frac{G_F^2 f_\pi^2}{4\pi^2} \frac{E}{|\boldsymbol{q}|} uv G_A^2 \frac{d\sigma(\pi^\circ N \to \pi^\circ N)}{dt}. \quad (7)$$

For isoscalar targets $d\sigma(\pi^+ N \to \pi^+ N)$ equals $d\sigma(\pi^\circ N \to \pi^\circ N)$.

In the widely used Rein-Sehgal (RS) model [11] the kinematical factors on the right-hand side of (7) have been evaluated for $Q^2 = 0$ resulting in the simpler expression

$$\frac{d\sigma^{\rm NC}}{dQ^2 dy dt} = \frac{G_F^2 f_\pi^2}{4\pi^2} \frac{1-y}{y} G_A^2 \frac{d\sigma(\pi^\circ N \to \pi^\circ N)}{dt} \quad (8)$$

for the cross section.

We have compared (7) to (8) by performing a Monte Carlo integration using the illustrative ansatz

$$\frac{d\sigma(\pi N \to \pi N)}{dt} = \sigma_0 b e^{-bt}, \quad (9)$$

with energy independent coefficients $\sigma_0 = 80$ mb and

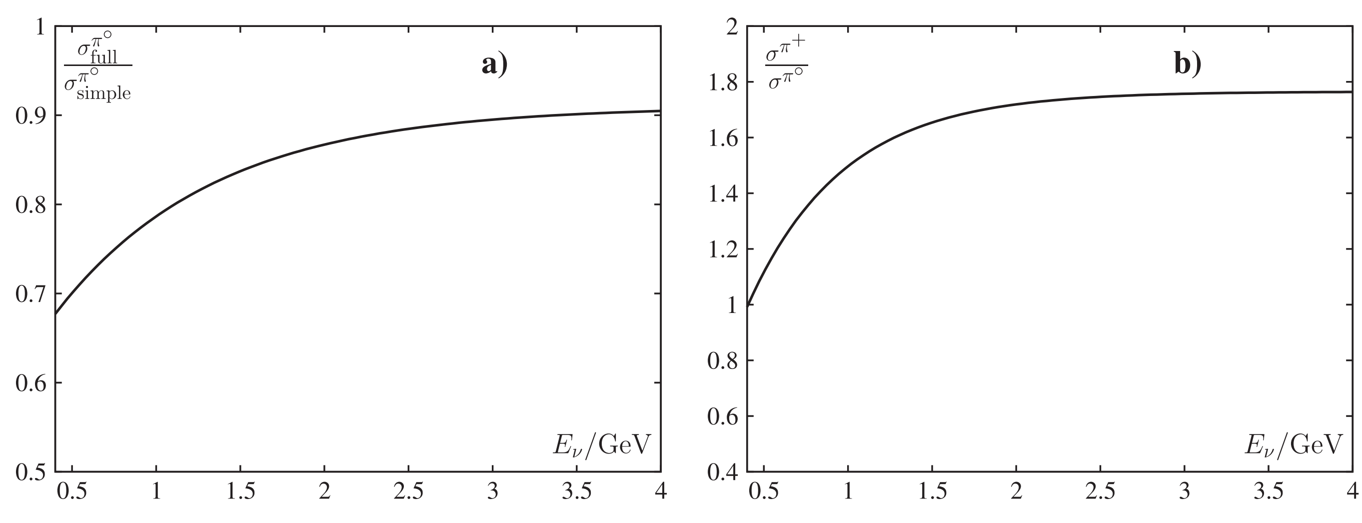

FIG. 1. (a) Ratio $\sigma^{\pi^0}_{\rm full}/\sigma^{\pi^0}_{\rm simple}$ of the integrated cross sections of (7) and (8) versus the energy of the incoming neutrino. (b) Ratio $\sigma^{\pi^+}/\sigma^{\pi^0}$ of the integrated cross sections (6) and (7) versus the energy of the incoming neutrino.

$b = 45\ \mathrm{GeV}^{-2}$ for elastic π° or π^+ scattering off carbon nuclei. (This ansatz will be motivated in the next section.)

The result of the integration is shown in Fig. 1(a) where the ratio $\sigma^{\pi^0}_{\mathrm{full}}/\sigma^{\pi^0}_{\mathrm{simple}}$ is plotted versus the energy of the incoming neutrino. Here $\sigma^{\pi^0}_{\mathrm{full}}$ stands for the integral of (7) and $\sigma^{\pi^0}_{\mathrm{simple}}$ for the integral of (8). This figure demonstrates that at low energies the neutrino cross section is substantially reduced by using the complete kinematical factors of (7).

We also compared π^+ production to π° production by integrating (6) and (7) using identical form factors and pion nucleus cross sections. Apart from the factor $\cos^2\theta_C$ one would naively expect a ratio of 2 according to the ratio of the pion decay constants. Figure 1(b) shows however a remarkable violation of this isospin symmetry due to lepton mass effects contained in (6). The dominant cause for the variation observed in the figure is the reduced phase space of CC reactions. The Adler screening factor reduces the CC cross section by further 10% at $E = 0.6$ GeV and 4% at $E = 2$ GeV. The influence of the numerical value of m_A on the cross section ratio is negligible. Even setting $G_A = 1$ changes the ratio by less then 2%.

IV. THE ELASTIC PION CARBON CROSS SECTION

In [11] a model has been presented which calculates elastic pion nucleus scattering from pion nucleon scattering via

$$\frac{d\sigma(\pi N \to \pi N)}{dt} = A^2 \frac{d\sigma_{\mathrm{el}}}{dt}\bigg|_{t=0} e^{-bt} F_{\mathrm{abs}}. \tag{10}$$

Here the pion on the left-hand side can be charged or neutral. The differential elastic pion nucleon cross section in forward direction on the right-hand side is determined via the optical theorem (neglecting a possible real part of the scattering amplitude)

$$\frac{d\sigma_{\mathrm{el}}}{dt}\bigg|_{t=0} = \frac{1}{16\pi}\left(\frac{\sigma^{\pi^+ p}_{\mathrm{tot}} + \sigma^{\pi^- p}_{\mathrm{tot}}}{2}\right)^2 \tag{11}$$

and the slope b of the exponential t distribution is taken from the optical model relation

$$b = \frac{1}{3} R_0^2 A^{2/3} \tag{12}$$

with e.g. $R_0 = 1.057$ fm. F_{abs} describes the average attenuation of a pion emerging from a sphere of nuclear matter with radius $R_0 A^{1/3}$ resulting in

$$F_{\mathrm{abs}} = \exp\left(-\frac{9A^{1/3}}{16\pi R_0^2}\sigma_{\mathrm{inel}}\right) \tag{13}$$

with

$$\sigma_{\mathrm{inel}} = \frac{\sigma^{\pi^+ p}_{\mathrm{inel}} + \sigma^{\pi^- p}_{\mathrm{inel}}}{2}. \tag{14}$$

As an example we calculate the total elastic pion carbon cross section via

$$\sigma_{\mathrm{el}}(\pi^{12}C \to \pi^{12}C) = \frac{A^2 F_{\mathrm{abs}}}{16\pi b}\left(\frac{\sigma^{\pi^+ p}_{\mathrm{tot}} + \sigma^{\pi^- p}_{\mathrm{tot}}}{2}\right)^2. \tag{15}$$

The total pion nucleon cross sections are available as computer readable files [12]. The data were fitted by a superposition of Breit Wigner functions and a Regge inspired term $a_0 + a_1/\sqrt{|\boldsymbol{p}_\pi|}$ with $|\boldsymbol{p}_\pi|$ denoting the pion laboratory momentum. A similar fit to the elastic cross sections finally yields $\sigma_{\mathrm{inel}} = \sigma_{\mathrm{tot}} - \sigma_{\mathrm{el}}$ which is used for calculation of F_{abs}.

The dotted line of Fig. 2 shows the result. The fact that this cross section is derived from a simple (classical) ansatz expressed by (10) and (13) raises doubts about its validity as a description of pion nucleus scattering in the resonance region. An alternative approach to coherent pion nucleus interaction based on the Glauber model was proposed by Bel'kov and Kopeliovich [13]. Its numerical results were similar to those in the RS model at least at high energies. The experimental groups use detailed Monte Carlo routines to simulate the scattering and absorption of the pion inside the nucleus [14,15].

Following the PCAC route we have tried to circumvent the uncertainties in modeling nuclear processes by direct appeal to data on pion nucleus elastic scattering; see also [16]. For carbon targets this can be done easily because π^+ and π^- data on differential and total cross sections exist for pion kinetic energies T_π from 30 to 870 MeV. They have been subjected to phase shift analyses yielding up to 21 complex phase shifts per energy [17]. Neglecting electromagnetic effects these phase shifts can be used to compute

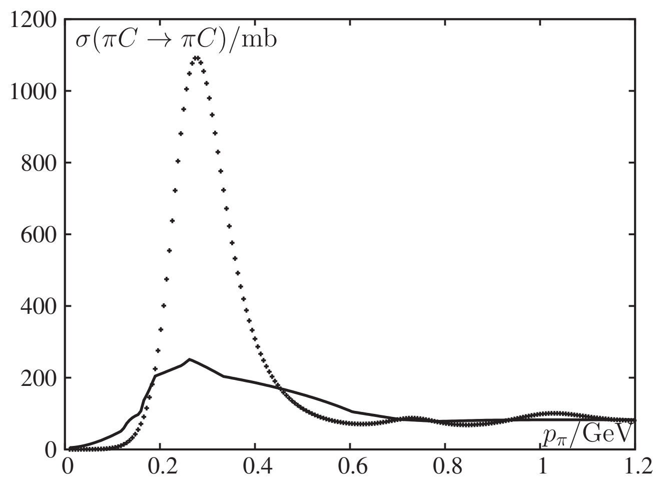

FIG. 2. Total elastic pion carbon cross section versus pion laboratory momentum. The dotted line represents the Rein-Sehgal model according to (15) and the solid line is derived from pion carbon data as explained in the text.

TABLE I. Coefficients A_1, b_1 of Eq. (16).

T_π (GeV)	A_1 (mb/GeV^2)	b_1 (1/GeV^2)
0.076	11 600	116.0
0.080	14 700	109.0
0.100	18 300	89.8
0.148	21 300	91.0
0.162	22 400	89.2
0.226	16 400	80.8
0.486	5730	54.6
0.584	4610	55.2
0.662	4570	58.4
0.776	4930	60.5
0.870	5140	62.2

the elastic strong interaction cross section $d\sigma_{el}/dt$ for pion carbon scattering in a straightforward manner.

The phase shifts accurately reproduce even tiny effects like secondary peaks in the angular distribution. We have checked that except for the lowest two kinetic energies of 30 and 50 MeV it suffices to parametrize the cross section by the simple ansatz

$$\frac{d\sigma_{el}}{dt} = A_1 e^{-b_1 t} \tag{16}$$

with energy dependent coefficients A_1, b_1 which are listed in Table I. For energies between the measured data points these coefficients are linearly interpolated which is the reason for the zigzag structure of the solid line in Fig. 2. It is obvious that σ_{el} from pion carbon data is much below the RS model in the resonance region. At the same time one observes that as $|\boldsymbol{p}_\pi|$ approaches 1 GeV, the two curves become very similar with $\sigma_{el} \approx 80$ mb. This finally justifies the ansatz (9). It also suggests that the RS hadronic model fails in the region of the Δ resonance, but may be a valid description at higher energies.

V. RESULTS

We are now ready to integrate the cross section (6) for the two different models of pion carbon scattering discussed in the last section. The results are plotted versus the neutrino energy in Fig. 3(a) for π^0 production and in Fig. 3(b) for π^+ production. In obtaining the lower curves the empirical pion carbon cross sections were calculated by assuming the coefficients in the last line of Table I to be valid up to $T_\pi = 1.7$ GeV. An error of 30% in this assumption results in a cross section error of 6% at $E = 2$ GeV.

The curve using carbon data is a factor of 3 to 2 below the curve obtained by applying the RS hadronic model. Cross sections for NC and CC coherent single pion production on carbon have also been calculated using an ansatz based mainly on the microscopic process $\nu p \to \mu^- \Delta^{++}$ and its modification in the nuclear environment [18–21]. (For an early reference to this subject see [22].) Remarkably our calculations agree well with the corresponding results given in [20,21] based on a very different approach to coherent neutrino scattering. The predicted cross sections of [16] depend sensitively on a cut parameter ξ. Referring to footnote 41 of [16] with $\xi = 1$ the results are close to the ones obtained in this paper. The differential cross sections $d\sigma/dQ^2$ or $d\sigma/d\cos\theta_l$ are more sensitive to details of the theoretical models. We give in Fig. 4 our prediction for $d\sigma/dQ^2$ at a neutrino energy of 1 GeV. For the CC reaction a pronounced dip in forward direction is seen which is mainly due to the Adler screening factor contained in the rectangular brackets of (7).

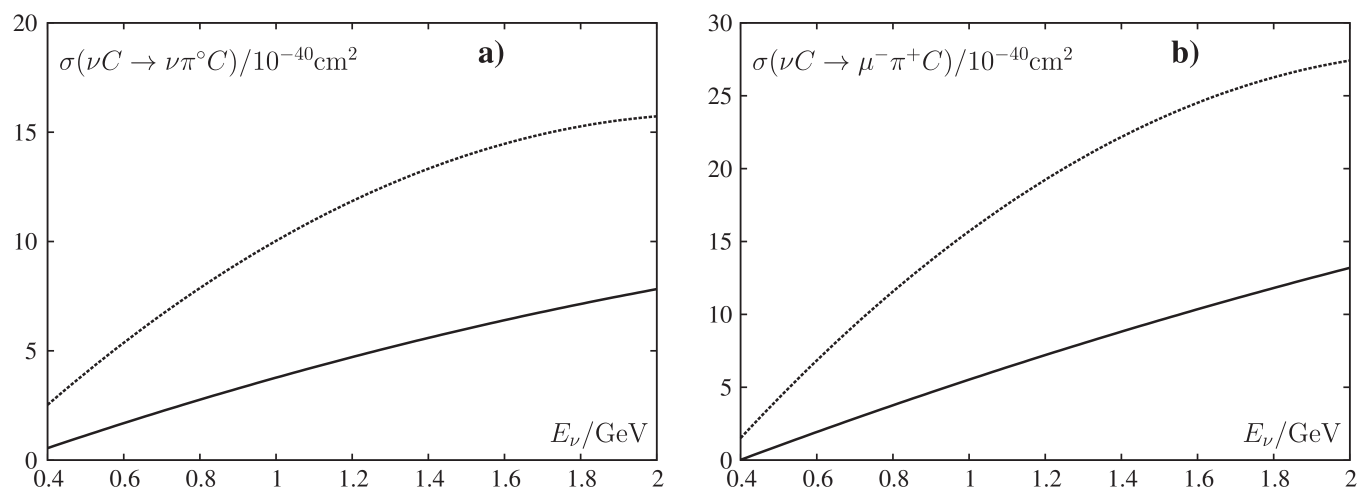

FIG. 3. Cross section per nucleus of coherent π production by neutrinos off carbon nuclei, (a) NC reaction $\nu_\mu + {}^{12}\text{C} \to \nu_\mu + {}^{12}\text{C} + \pi^\circ$, and (b) CC reaction $\nu_\mu + {}^{12}\text{C} \to \mu^- + {}^{12}\text{C} + \pi^+$. The results in units of 10^{-40} cm^2 are plotted versus the neutrino energy in GeV. The upper curve is calculated using the hadronic RS model, the lower curve using our parametrization of pion carbon scattering data.

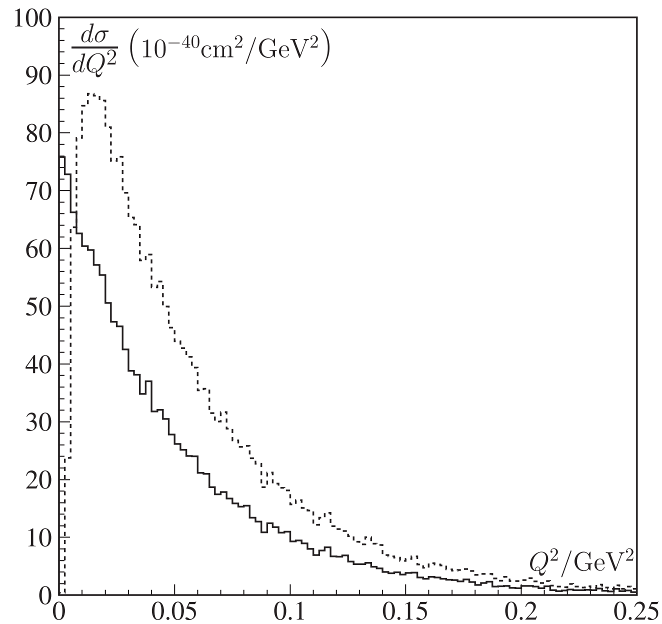

FIG. 4. Differential cross section $d\sigma/dQ^2$ per nucleus for coherent single pion production off carbon nuclei. The data are obtained by integrating (7) using carbon data for $\sigma_{\pi N}$. The neutrino energy is 1 GeV. The solid line is for the NC reaction and the dashed line for the CC reaction.

Comparing to experimental results we first discuss π^+ production. At a neutrino energy of 1.3 GeV the K2K experiment [23] has placed an upper limit of 0.60×10^{-2} on the cross section ratio of coherent pion production $\sigma_{\rm coh}^{\pi^+}$ to the total CC cross section $\sigma_\nu^{\rm CC}$ at 90% confidence level (CL). Using the K2K cut in the muon momentum ($p_\mu > 0.45$ GeV) our prediction is $\sigma_{\rm coh}^{\pi^+} = 0.62 \times 10^{-40}$ cm^2 per nucleon. With $\sigma_\nu^{\rm CC} = 107 \times 10^{-40}$ cm^2 as quoted in [23] we obtain $\sigma_{\rm coh}^{\pi^+}/\sigma_\nu^{\rm CC} = 0.58 \times 10^{-2}$ which is consistent with the K2K result.

The SciBooNE experiment [15] has measured an upper limit of 0.67×10^{-2} at 1.1 GeV (90% CL) on the same cross section ratio. Assuming the $\sigma_\nu^{\rm CC}$ value quoted in [23] to scale linearly with E we obtain $\sigma_\nu^{\rm CC}/E = 82.3 \times 10^{-40}$ cm^2/GeV in the low energy regime and $\sigma_{\rm coh}^{\pi^+}/\sigma_\nu^{\rm CC} = 0.58 \times 10^{-2}$ taking $\sigma_{\rm coh}^{\pi^+}$ from Fig. 3(b). At 2.2 GeV the SciBooNE upper limit is 1.36×10^{-2} to be compared with a calculated ratio of 0.68×10^{-2}. The latter value requires a slight extrapolation of Fig. 3(b).

The improved model for coherent pion production is thus in good agreement with the π^+ measurements. It is possible that the lower cross section for coherent π^+ production also reduces the discrepancy at small Q^2 in CC reactions noted by the MiniBooNE Collaboration [24].

Turning to π° production the published coherent fraction $\sigma_{\rm coh}/(\sigma_{\rm coh} + \sigma_{\rm incoh})$ of the MiniBooNE experiment [14] is $(19.5 \pm 1.1({\rm stat}) \pm 2.5({\rm sys}))\%$ at a nominal neutrino energy of 1.2 GeV. Using the Rein-Sehgal model for incoherent production [25] and our present calculation of the coherent cross section we get a coherent fraction of $\approx 5\%$ which is much below the experimental findings. It would be interesting to see if there is a dependence of the experimental fraction on the details of the coherent model used in the analysis. It should be stressed that the theoretical prediction for coherent scattering covers only reactions where the nucleus stays intact and does not break up during interaction. The data on pion carbon scattering [17] show that nearly two-thirds of the cross section is inelastic. Our estimate of the error in the theoretical prediction (taking account of the model dependence of $\sigma_{\rm incoh}$, the extrapolation in Q^2, the interpolation in Fig. 2, and the neglect of the transverse contributions $\sigma_{R,L}$ estimated in [16]) is 20%.

The total pion nucleus cross section scales $\sim A^{2/3}$ at least for small atomic numbers A [26]. If the ratio of elastic to total cross section depends only weakly on A we then expect the elastic cross section also to scale proportional to $A^{2/3}$. Using the RS hadronic model the ratio of the cross sections for aluminum ($A = 27$) and carbon is 1.68 for $E = 2$ GeV corresponding to a scaling law $\sim A^{0.63}$. The algorithm presented in the preceding section can thus probably be simply extended to other light nuclei. Applying the $A^{2/3}$ scaling law we obtain for aluminum $\sigma_{\rm coh}^{\pi^\circ} = 13.3 \times 10^{-40}$ cm^2 at $E = 2$ GeV which agrees within errors with the experimental value of $(27 \pm 17) \times 10^{-40}$ cm^2 measured by the Aachen Padua experiment [27].

For neutrino energies >2 GeV the calculation using carbon data is not applicable because a large part of the integration then needs carbon data for $|\boldsymbol{p}_\pi| > 1$ GeV which are not available. The fact that the RS prediction for pion carbon scattering overlaps with the empirical result in the region $|\boldsymbol{p}_\pi| > 0.7$ GeV (Fig. 2) may be a hint that the RS model begins to be a valid description of coherent pion production beyond the resonance region. Such a view would explain the impressive agreement of the RS model with the large body of data on coherent pion production in the energy domain $E = 2 \ldots 100$ GeV, as documented, for example, in [28].

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